STATISTICAL DESIGN OF AGRICULTURAL EXPERIMENTS*

BY

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1. INTRODUCTION

Friends, it is a great pleasure for me to be with you this morning. I believe that we are meeting at an important point in the life of the Society. The Society has just completed 30 years of its existence, and I believe it is about to enter a new and more active and fruitful era. During the life span of the Society so far, the population of the country has approximately doubled. The number of new mouths to feed increases every year. Right now, it is approximately one Australia per year; but soon we may be adding one Canada every year. To cope up with this, agricultural research and development is needed at an unprecedented rate. Thus the importance of an organisation like ours cannot be exaggerated. Our new Government has very wisely increased the outlay in the agricultural sector, and is vigorously introducing reforms and various schemes designed to promote growth in all directions of agricultural activity. For the survival of the country, such a move is absolutely necessary. However, unfortunately, it is not sufficient by itself. In other words, I hope I did not give you the impression that there exists a way of promoting agricultural reforms, including research and development, which would result in a level of food production which could, for ever, top the population growth at the present rate, because there doesn't. Thus, whatever positive development is possible in the field of agriculture in India, will have to be done, indeed done in a hurry. I, therefore, foresee an era of increased pressure on the scientists in the agricultural sciences for help. In such a situation, agricultural experimentation both on an extensive and intensive scale will be called for. Research on the methodology for the statistical design of such experiments therefore continues to remain very important. This is

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particularly so because of the limited financial resources. It is well known, at least in competent statistical circles, that experimental scientists who ignore good statistical planning for their experiments, squander tax-payers' money. A country like India can ill-afford to do so, and emphasis on research in, and application of, statistical planning is particularly important here.

In what follows, I propose to discuss in brief many areas of research in the field of statistical design, which are particularly important from the point of view of application to agricultural experimentation. Though, occasionally, our remarks will be pertinent in general, the discussion will be in the context of agronomic experiments like varietal trials, fertilizer trials etc.

2. Some Areas of Experimental Design

We first discuss certain broad areas of design, which are relatively new or have to be looked at from a new angle, and furthermore, which are relevant to good statistical planning of agricultural experiments.

(a) *Factorial designs*

We consider these first, since later discussion will be frequently centered on this subject. Consider an $s_1 x s_2, \ldots x s_m$ factorial experiment, in which there are m factors, the ith one having s_i levels. There are three models which are in common use. I shall call them *(i)* local effects model, *(ii)* global effects model, and *(iii)* mixture model. For brevity, I shall illustrate these for $m=3$. Suppose (a_i, b_j, c_k) is a treatment combination in which a, b, c denote factors, and i, j, k, the corresponding levels, let $t\ (a_i, b_j, c_k)$ denote its "true effect". Then, for example, under the local effects model we may assume that

$$t\ (a_i; b_j, c_k) = A_i + B_j + C_k + (AB)_{ij} + (AC)_{ik} + (BC)_{jk},$$

where A_i denotes the effect of the ith level of the a-factor, $(AB)_{ij}$ the interaction beween a_i and b_j, etc. Often, some or all the interaction terms are assumed negligible. Designs which are suitable for use under this model are called multi-dimensional designs. An important sub-class of these is that of the so-called 'row-column' designs. These are useful for varietal trials, when interest lies in two-way elimination of heterogeneity. Some important studies in this direction have been made by Kiefer, Seiden, etc. who studied generalised Youden designs, particularly in the context of variance-oriented optimality. Another interesting development is that on F-squares, by Hedayat, and others. In multi-dimensional designs, the problem of estimability of parameters is related to the concept of 'connectedness'. In the local effects model without interaction necessary and sufficient conditions for the estimability of all parameters were given by

Srivastava and Anderson. Further characterisation of these in graph-theoretic terms should be quite useful, particularly if the technique results in some kind of a picture which can be visually inspected and evaluated.

The global effects model is sometimes called the classical factorial model. This is the model used, for example, by Bose and Kishen, or Fisher for the development of the classical theory of confouding. There are two sub-classes under this model, depending on whether the interactions are defined under (*a*) the product definition (which uses orthogonal polynomials), or (*b*) the geometric definition (for the case when $s_1 = \ldots = s_m = s = a$ prime number or a power of a prime number); which uses Galois fields and geometries. The definition of the main-effects is the same in the two cases. Also, the two definitions are identical for che case of 2^m factorials. In the general case, the set of interactions (between a given set of factors) under one definition can be obtained by making an orthogonal transformation of the corresponding set of interactions under the other definition. Details of the two definitions will be found in, for example, Bose and Srivastava (Sankhya 1964), Bose (Sankhya 1947), etc.

The parameters under the global model are related to those under the local effects model, but the interpretations of the main effects and the interactions etc. are fundamentally different in the two cases.

The mixture model is one in which some factors are treated as under the global model, while the remaining factors and their interactions with the factors of the first set, are treated as under the local effects model. A simple example would be that of a confounded symmetrical factorial design of the Bose-Kishen type. Here, the blocks constitute a factor of the local effects type, and the treatments of the factorial involved are treated as under the global (geometric) effects model. If block-treatment interactions are assumed, these of course are treated under the local effects model.

Needless to say, before embarking on any multifactorial experiment, it is of prime importance to consider exactly what kind of model will be used in analysing the data. Failure to do so, which unfortunately happens more often than is generally realized, results in a lack of or total loss of information on certain comparisons which would have been of interest. However, this does not mean that once the experiment has been finished and the data collected, the analysis has to be done only under the pre-planned model. Usually, it is more insightful to analyse the data in many different ways, some of which could even be partly or wholly informal.

(*b*) *Designs with 2-dimensional blocks*

Briefly speaking, these are designs of the incomplete block type in which, in each block, it is possible to eliminate heterogeneity in two directions. Thus, ordinary 'row-column' designs commented upon in the previous sub-section are a special case of these. However, in those designs, there is generally only one large (square or) rectangle, and various treatments are applied to units which are represented by the cells of this rectangle. For the new designs considered in this sub-section, this restriction is waived. Thus, in general, we have a set of b blocks, whers $b \geqslant 1$. Let v denote the number of treatments. A well-known example of the new class of designs is a lattice square.

Take, for example, a complete Lattice Square design with $v = 9$. In this case, we have 2 replications for each treatment, heterogeneity is eliminated in two directions, and it is well-known that every pair of treatments is compare with the same accuracy. In general, when $v = t^2$, complete balanced is achieved in $(t+1)$ replications when t is even, and in $\frac{1}{2}$ $(t+1)$ replications when t is odd. Thus, for large v, the number of replications for complete balance will be large. Hence, there is a need for obtaining designs in which the average number of replications (in case each treatment is not replicated the same number of times), is relatively small. Now, for this purpose, occasionally we may have to sacrifice complete balance. However, this is not too crucial in many cases, since some kind of partial or semi-balance may be satisfactory. Some work in this direction by the author is currently in progress. Below, we give two examples. An other example will be found in the paper by Srivastava (1976), on optimal search designs, published in *Statistical Design Theory* Volume II (Editors: Gupta and Moore) where the idea of such designs was first presented.

We present a class of cyclic designs for any given v. Choose 4 distinct numbers, say a_1, a_2, a_3 and a_4, satisfying

$$o \leq a_1,\ a_2,\ a_3,\ a_4 \leq v-1.$$

Take b two-dimensional blocks as follows.

In the uth block ($u=0, 1, \ldots\ldots, v-1$), put the treatments as indicated in the square below :

a_1+u	a_2+u
a_3+u	a_4+u

If, for any u, the number a_i+u is larger than v, then we substitute it by a_i+u-v. The a_i can be chosen, for example, from the view point of overall efficiency or other variance-oriented optimality considerations. The number of replications is obviously 4.

We next consider an example from a large class of designs with two dimensional blocks, which are obtainable from ordinary BIB or PBIB designs. Thus, consider a BIB or PBIB design with parameters b_0, v_0, r_0, k_0, where v_0 denotes the number of treatments, b_0 the number of blocks, r_0 the number of replications of each treatment, and k_0 the block size. Now, consider a design of the new type with $v=v_0$, and $b=b_0$, constructed as follows. Make square block of size $k_0 \times k_0$. The v_0^2 treatments can be denoted by pairs (i, j) where i, $j=1, 2, \ldots, v_0$. Now suppose that ith block of the design contains treatments $(p_1, p_2, \ldots, p_k)$. Then we take a complete set of lattice squares of size $k_0 \times k_0$ which has the treatments (i, j) where i, $j=p_1$, $p_2, \ldots, p_{k0}$. It will be seen that the number of replications is not the same for each of v_0^2 treatments. If the original design is a BIB design, then every treatment of the form (i, j) will be replicated $r_0 q$ times, where $q=\frac{1}{2}(k_0+1)$, or (k_0+1), according as k_0 is odd or even. Also, the other treatments will be replicated $1_0 q$ times where 1_0 is a constant which for the BIB equals $r_0(k_0-1)/(v_0-1)$. An example of such a design with $v_0=4$, $r_0=3$, $k_0=2$, $b_0=6$ is given below :

1	2	1	6	1	5
5	6	2	5	6	2
1	3	1	9	1	11
9	11	11	3	3	9
1	4	1	13	1	16
13	16	16	4	4	13
6	7	6	10	6	11
10	11	11	7	7	10
6	8	6	14	6	16
14	16	16	8	8	14
11	12	11	15	11	16
15	16	16	12	12	15

The new class of designs described in this sub-section has many nice features. They seem to be useful for reducing the number of replications, having at the same time a satisfactory amount of balance. Also the number of treatments is relatively large. Of course, on the top of all this, they enable us to eliminate heterogeneity in two directions within each block and are, therefore, very sensitive. Finally, small block size makes the row-column interaction less likely to exist.

(c) Optimal designs

In the past two decades, considerable work has been done in the field of optimal designs. The word 'optimal' here refers to variance-based optimality. In other words, suppose that $\underline{g}$ denotes a vector of parameters, of interest, and that $\hat{\underline{g}}_D$ denotes its estimate when a given design D is used. Let V_D denote the variance-covariance matrix of $\underline{g}_D$. The design D is chosen so as to minimise V_D in some sense. Thus, for example, the determinant, trace, and the largest root optimality criteria refer to the minimization respectively of the determinant, the trace, and the largest root of V_D. Usually, minimization of V_D is done under the variation of D within some class of designs. For example, it may be the class of designs, such that each design within the class involves the same number of observations (or, more generally, the same cost). Some times, further appropriate restrictions may be placed on the class of designs from which the optimal one is to be selected. For example, we may require the design to be 'balanced' in some sense.

A factorial design with m factors (under the global model) is said to be of resolution $(2h+1)$ if it allows the estimation of all factorial effects involving h factors or less, assuming that all effects involving $h+1$ or more factors are negligible. Trace-optimal balanced resolution $(2h+1)$ factorial designs of the 2^m series have been obtained during the past decade by Srivastava, Chopra, Yamamoto, Shirakura, Kuwada etc. Considerable work has also been done on designs of even resolutions which, we do not consider here for lack of space. Work on the 3^m case is currently being conducted by Srivastava, Kuwada, Ariyaratna, and possibly others.

A great deal of work is also going on for the situation when the factors are continuous. Such designs are sometimes called response-surface designs. However, it is widely agreed upon that for agricultural experiments it is more suitable to take factors with a small discrete set of levels. We shall therefore not dwell on the continuous case any further, although in general it is very important.

(d) Search Designs

We first recall the search linear model. Let $\underline{y}$ be a vector of N observations, $A_1\,(N \times n_1)$, $A_2\,(N \times n_2)$ known matrices, σ^2 a known or unknown constant, $\underline{g}_1\,(n_1 \times 1)$ and $\underline{g}_2\,(n_2 \times 1)$ vectors of parameters, such that

$$\text{Exp}\,(\underline{y}) = A_1\underline{g}_1 + A_2\underline{g}_2,\ \text{Var}\,(\underline{y}) = \sigma^2 I_N$$

Further, suppose that $\underline{g}_1$ is completely unknown while on $\underline{g}_2$ we have partial information available. In particular, suppose that there exists an integer k (known or unknown), such that at most k elements of $\underline{g}_2$ are non-negligible. The problem is to search the non-negligible elements of $\underline{g}_2$, and draw inferences on these and on the elements of $\underline{g}_1$. Such a model is called a 'search linear model'. Clearly, the case $k=0$ gives an ordinary linear model as a special case. On the other hand, since it is not known as to which elements of $\underline{g}_2$ are non-negligible, the search linear model is not exactly a special case of the ordinary linear model.

A great deal of work has been done in recent years on different subjects in relation to this model. One such area is that of 'search designs'. Thus, consider a 2^m factorial. Let $\underline{g}_1$ correspond to the general mean, main effects, and two-factor interactions, while $\underline{g}_2$ corresponds to the three-factor and higher effects. Suppose $k=1$. Then a set of treatments T are said to constitute a design of resolution 5·1, if it allows us to search the non-negligible element of $\underline{g}_2$ and estimate it and also $\underline{g}_1$. Such designs exist, and are not large in size. A class of such designs has actually been obtained by Srivastava and Ghosh (Communications in Statistics, 1977).

Work has also been done on methods of search of the unknown non-negligible parameters in search linear models. Some Monte Carlo and other work in this connection shows that the probability of correct search is satisfactorily high.

Unless one is absolutely sure that $k=0$ in a given situation, the search designs constitute a uniform improvement over the classical ones. Since agricultural experimentation is a very important area of application of discrete factorial designs, one would hope that search designs will be increasingly used in this field in most situations.

(*e*) *Multiresponse designs*

In most agricultural experiments, more than one response or characteristic is measured on each experimental unit. However, most of the time the experiment is planned as if only one response was under consideration. A design specially suited for a multi-response experiment is called a 'multiresponse design'. The data resulting from such an experiment should be analysed using multivariate analysis of variance and other suitable techniques of multivariate (including univariate) analysis. A humble beginning has been made on the theory of multiresponse designs and their analysis, some of the work having been reported in Roy, Gnandesikan and Srivastava (1970, Analysis and design of Multiresponse Experiment, Pargamon Press). However, much more still needs to be done.

An important study which could and should be made from the data of such experiments is the relationship between the various response variables. Suitable designs for this purpose will need to be evolved. Notice that the word design has two aspects in the present context. One is the question of deciding which treatment should be applied to which unit. The other aspect is to decide for any given unit, whether we want to measure each response on it or not. A basic question, essentially related to the above, concerns which responses to study, and how intensively a particular response should be studied. This is in general a rather difficult problem, since the number of possible responses that could possibly be measured is usually immensely large. However, a little reflection will reveal that this problem is at the heart of the matter, and any insight in this direction should be greatly rewarding.

3. Further Research Needs

As mentioned earlier, generally speaking, agricultural experimentation is such that it is more appropriate to use factors with a discrete set of levels. Even when a factor is basically continuous (like the amount of a fertilizer), only a few intelligently chosen levels should be included in the experiment. Thus, in most situations, discrete designs (*i.e.*, those in which each factor has discrete levels) are necessary.

Another important feature of the agricultural field is that experimentation is relatively slow, since raising a crop or animals takes months or years. This is in contrast to the physical or industrial sciences, where experiments can usually be done quite fast, which allows a relatively large number of experiments to be done successive ly, within a reasonably short period of time. Because of this, the methods of classical sequential design and the theory of stopping rules etc. are largely out of place here.

What is needed is a good essentially single-stage experiment, which is to be planned in the light of past experience (including one or more experiments in the same series).

The above comments make it clear that search designs have a very important role to play in agricultural experimentation. The fact that we need good essentially single stage designs makes them specially suitable. Furthermore because of the discrete nature of experiments, it is clear that search designs are needed in the context of multifactorial experiments. All the three models namely the local, the global and the mixture models will be arising frequently. Thus, research in all these directions will be potentially important from the applied point of view.

I may however add that the new search designs to be developed need not be just the resolution 5.1 designs mentioned earlier, or simple extensions of these. In an institution like the IASRI, specific designs could be developed for the type of problems at hand. In other words, the vectors $\underset{\sim}{g}_1$ and $\underset{\sim}{g}_2$ should be determined in the light of the past experience. In general, all factorial effects, which from the available experience are expected to be non-negligible (even with a relatively low probability) should be thrown into $\underset{\sim}{g}_1$. Once $\underset{\sim}{g}_1$ and $\underset{\sim}{g}_2$ are determined, a safe but not too large value of k should be selected. Finally, a search design should be obtained for this purpose. This part may be quite non-trivial in general, and so I would like to urge the more theoretical researchers to consider developing alogrithms for obtaining search designs. This may have to be combined with other aspects like nuisance parameters etc. not discussed here.

The above should also be combined with the variance-oriented optimality theory. Some beginnings have been made in the author's paper on 'optimal search designs', mentioned earlier.

One may ask as to how to proceed until the theoretical problems involved in the above have been successfully attacked. Well, the answer has to be to use the best design that is available. For example in certain situations, instead of wasting money by using a very large orthogonal factorial design of the classical type, one could use a suitable optimal balanced design (having a desirably small number of runs N). Again, in preference to the last mentioned design, one may use a search design if it is available. Unfortunately, however, for general $\underset{\sim}{g}_1$, even ordinary optimal designs (*i.e.* those with $k=0$) are not available.

The need for new confounded designs, though they usually require a large number of experimental units, still continues to exist. Some of the more important requirements appear to be (*a*) reduction of the number of replications, and (*b*) development of designs for various block sizes, which may even be unequal. Of course, at the same time there should be a satisfactory amount of balance. Thus, for example, one may need a suitable confounded design of the 3^3 type when we have a given number of blocks of size 5, 6, and 7, the requirement being to waste as few blocks as possible. Another important problem is to look at specific contrasts as in the work of Pearce, Calinski, etc.

For asymmetrical factorials, of course, further work needs to be done. A beginning was made by Kishen and Srivastava (Society's Journal, 1960), who for the first time developed a mathematical theory for the same, and showed how designs constructed earlier by trial and error were obtainable as a part of a rigorous theory. In the author's paper in the Panse Memorial Volume, there is a sentence in the first paragraph to the effect that a more powerful method was developed by M.N. Das (who, incidentally, was one of the Editors of the Volume). I am not the author of that remark, and do not subscribe to it.

Comments on further research needs on multiresponse designs have been made earlier. I may close by adding that at least in simple situations, we should try to develop designs which integrate various ideas put-forth above.